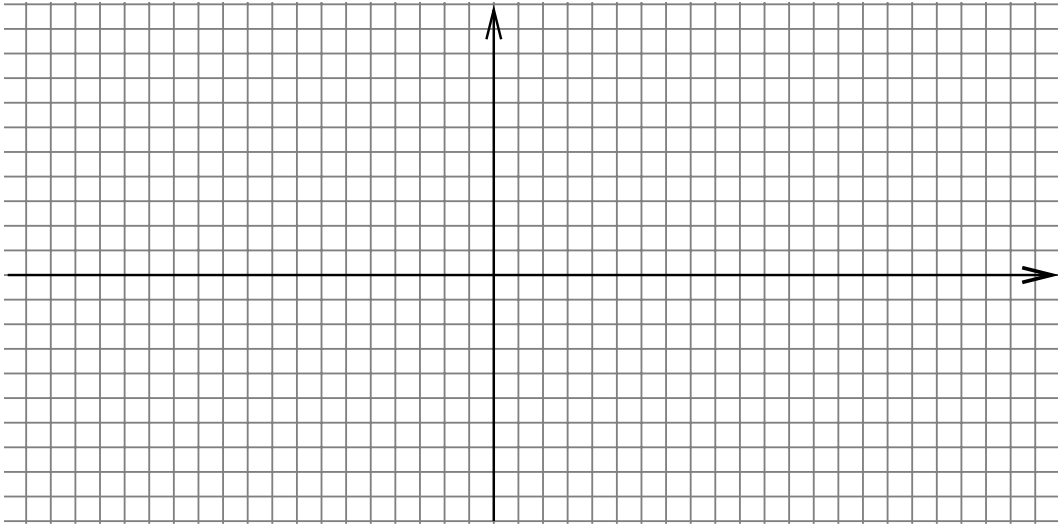### 6.3.4   The graph of the tangent function

Textbook Section 5.5

We now look at the graph of the tangent function. By contrast with $\sin(x)$ and $\cos(x)$, $\tan(x)$ is not defined everywhere:

Let's use this information to graph $\tan(x)$.

NOTES:

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6.4   Applications of sine, cosine and tan functions

6.4.1   The radius of the Earth

This is an application of the arc length formula, and of the relationship between degrees and radians. It is also a true story! *See textbook problem 123 section 5.1.*

Eratosthenes was a Greek scholar who lived in Cyrene around 200BC. On a particular day and time, he noticed that the Sun shone down exactly vertically into a well in Cyrene. He realized that, using this information, he could measure the radius of the Earth! To do so, he went to another city, Alexandria, which was 500 miles away and due North of Cyrene. Exactly 1 year (same day, same time) after his discovery of the well, he measured the angle the Sun made with the vertical, this time in Alexandria, and found that it was 7.2 degrees. How did he deduce the radius of the Earth?

6.4.2   Travel time between two points

This is an application of the right-angle triangle definitions of the sine, cosine and tangent functions. *See textbook problem 127 section 5.2.*

A jogger would like to go from point A to point B (see diagram below), across a river. She can jog 8 miles per hour on the road, but only 3 miles per hour on the beach.

How long does it take her to get from A to B, as a function of the angle θ? How long does it take when $\theta = 30°$? How long does it take when $\theta = 45°$?

6.5   Periodic functions

Textbook Section 5.6

6.5.1   Periodicity

DEFINITION:

Recalling that the angles are only defined within an additive factor of 2π:

we conclude that

EXAMPLE 1: Show that any function that is periodic with period p is also periodic with period np, where n is an integer number.

NOTE: Because of this, we often define:

- The *basic period*:

- The *harmonics*:

EXAMPLE 2: Show that the *basic* period of the tangent function is in fact π, not 2π.

EXAMPLE 3: What is the basic period of the function $\sin(2x)$?

6.5.2   Examples of oscillations in nature

Many real phenomena are prone to *oscillations*. An oscillation is defined a very regular periodic behavior, and can usually be expressed in terms of sine and cosine functions. Natural examples of oscillations in nature are:

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Oscillations are usually characterized by 4 numbers:

- The mean:

- The amplitude of oscillation:

- The period of oscillation:

- The phase of oscillation:

6.5.3   Modeling oscillations

To model these different types of oscillatory behavior, we can modify the basic trigonometric functions. Based on how the properties of the graphs of functions are changed, we see that

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But how do we change the period? To answer this question, let's remember that the period of function $\sin(2x)$ was π, not 2π. So, multiplying x by a number *within* the sine (or cosine) function changes the period of the oscillation. This is indeed a general rule:

To prove this:

6.5.4   General properties of the functions $f(x) = m + a\cos(bx + c)$ and $g(x) = m + a\sin(bx + c)$

We now generalize what we saw in the previous section: the functions $f(x) = m + a\cos(bx + c)$ and $g(x) = m + a\sin(bx + c)$ are oscillatory functions with the following properties:

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Note that the phase is often defined in different ways depending on applications, and also because sine and cosine themselves are actually the same function with a different phase (see previous sections).

EXAMPLE: What is the mean, period and amplitude of the following functions:

- $f(x) = 2 + 2\cos(2x + 2)$

- $f(x) = 3 + e\sin(\pi x)$

- $f(t) = \sin(2\pi t - 1)$

6.5.5   Application: tides

Santa Cruz surfers are usually very interested in finding out when the high- and low-tides are, at given locations along the coastal ocean. This information can be provided to them in the form of a function that records, for any time t, the height of the water $h(t)$ in feet. Tides have a period of approximately 12 hours. On November 15th, 2015, the mean water depth off Pleasure Point is 20ft. The amplitude of the tide is 3.7 ft, and the first high tide is at 1:30AM. What could the function $h(t)$ be, given that the time t is 1 at 1AM on November 15th? (Note: there are several possible answers).