

Chapter 6

Trigonometric functions

6.1 Degrees and radians

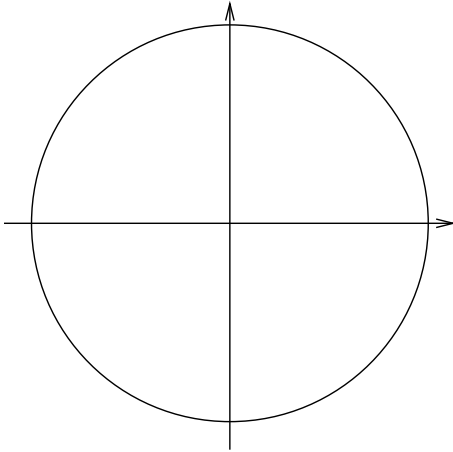
Textbook Section 5.1

There are two major ways of measuring angles in geometry: in *degrees* and in *radians*.

The degree measure was introduced historically in astronomy to measure the displacements of stars, and is based on the fact that there are approximately 360 days in a year (well, there are in fact 365.25 days in a year, but 360 conveniently divides nicely by 2, 3, 4, 6, 10, 12, ..., while 365.25 doesn't).

The radian measure is the one more commonly used in mathematics. It is based on the length of portions of a circle:

Based on this we have the correspondance:



To summarize, to go between radians and degrees and vice-versa,

By convention, in mathematics we also define a direction to an angle:

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Since the circle wraps around, an angle is always defined up to a value of 2π :

6.2 Right-angle triangles and basic trigonometric functions

6.2.1 Sine, cosine and tangent

Sine, cosine and tangent functions are usually defined through their association with right-angle triangles:

IMPORTANT CONSEQUENCES: From this diagram, we see that there are two very important formulae relating these three basic trigonometric functions to one another:

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6.2.2 Co-tangent, secant and cosecant

There are three more important functions to learn, defined as follows:

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These 6 functions altogether form the basic trigonometric functions you will need to know.

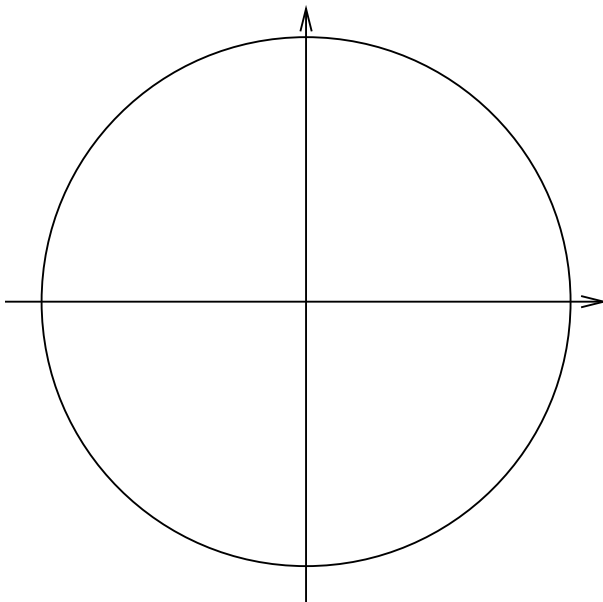
6.3 The unit circle, and the graphs of sine, cosine and tangent

Textbook Section 5.2

6.3.1 Construction of the unit circle

The unit circle is a wonderfully convenient way of *visualizing* the sine and cosine functions.

DEFINITION:



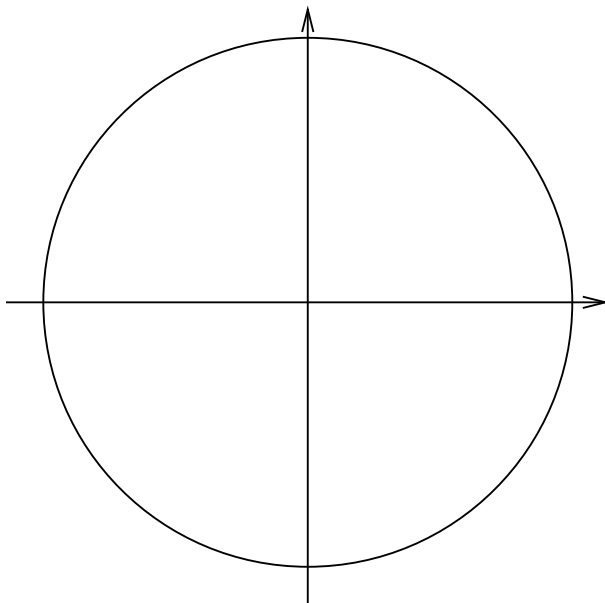
Based on this, we can already deduce some particular values of the sine, cosine and tangent functions:

6.3.2 Sine and Cosine of important angles

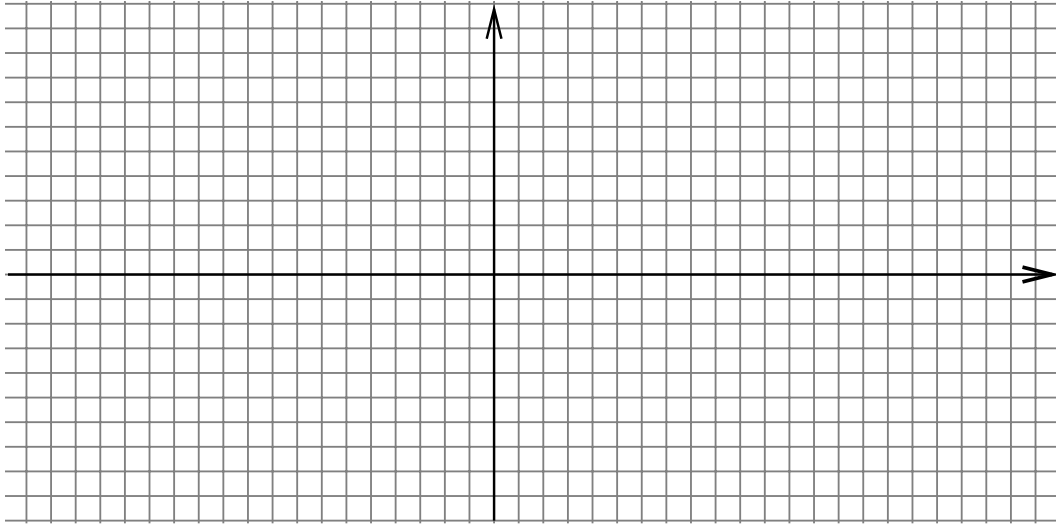
In addition to $\pi/2$, π , $3\pi/2$ and 2π , there are 3 important angles for which you need to know the sine and cosine of:

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Based on the unit circle, we can now find the sine and cosine of many other angles:



Finally, we can use this information to plot the sine and cosine functions:



6.3.3 What can we deduce from the graphs of $\sin(x)$ and $\cos(x)$?

Based on the graphs of $\sin(x)$ and $\cos(x)$, we see that

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