5.2 General logarithmic functions

Textbook Section 4.4

5.2.1 Definition and graph

Definition:

Graph:

Case 1: $a > 1$

Case 2: $0 < a < 1$

Domain of definition:

Universal property of logarithms:

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5.2.2 Examples of logarithms in common bases

The function $f(x) = \log_2(x)$ (logarithm in base 2)

The function $f(x) = \log_{10}(x)$ (logarithm in base 10)

Examples:

- $\log_{10}(1000) =$
- $\log_2(0.25) =$

5.2.3 The inverse relationships

Since the logarithm in base a is the inverse of the exponential in base a, we have the two fundamental relationships

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These relationships can be used to simplify expressions with exponentials and logs...
EXAMPLES:

- $\log_2(2^x) =$

- $\log_5(5\sqrt{5}) =$

- $\log_{10}(100^x) =$

- $3^{\log_3(2)} =$

- $10^{\log_{100}(2x)} =$

These relationships can also be used to prove important properties of logarithms...

5.2.4 Properties of the logarithms and examples of use

Textbook Section 4.5

The following rules apply for logarithms.

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To show why these formulas are true, we go back to the definition of the logarithm as an inverse, and use the properties of the exponentials: for instance, to show why the first formula is true write:

Similarly, we can also show why the second formula is true:

Then, using this, we can now see why the other ones are true as well:

EXAMPLES:

- Combine into one log expression: $\log_2(x^2-1)-\log_2(x+1)$

- Simplify $\log_2(8(x-2))$:

- Simplify $\log_{10}(100^{x+1})+\log_{10}\left(\frac{1}{5^x}\right)$

5.3 The natural exponential and the natural logarithm

There is one particular base called the *natural* base for exponentials and logarithm.

DEFINITION:

Remember that e is a real number, with value approximately equal to:

The reason why this peculiar base is important in mathematics will be explored in more detail in Calculus. However, for the moment, just accept the following property of the natural exponential:

Naturally, various function can be constructed from e^x:

DEFINITION:

PROPERTIES OF THE NATURAL LOGARITHM AND EXPONENTIAL: since these two functions are inverse of each other...

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5.4 Change of base rules

Textbook Section 4.5

There are a few manipulations of exponential and logarithms which involve changing bases with the natural base. It is important to master these changes of base formulae.

5.4.1 Changing from base a to the natural exponential

As it turns out, in Mathematics we very rarely use anything other than the natural exponential and logarithm (with the exception, perhaps, of the logarithm in base 10). Instead, whenever we have a real-life problem that is modeled by an exponential that is not the natural exponential (see the various applications we did earlier), we transform it into the natural exponential using the Change of Base Rules. To change base from a base a exponential to the natural exponential (and vice versa, if needed):

The reason why this works is simple:

EXAMPLES:

- $2^x =$
- $\left(\frac{1}{4}\right)^x =$

5.4.2 Changing from base a to the natural logarithm

The rule of change of base from a base a logarithm to the natural logarithm (and vice versa) is (see result above for example):

The reason why this works is simple too:

EXAMPLES:

- $\log_2(x) =$
- $\log_{\frac{1}{4}} x =$

NOTE: This formula, when applied to a, yields the obvious relationship

If you are not sure of your change-of-base formula, this is a good way of double-checking that the formula you remember is the correct one.

This change of base is particularly useful because most calculators only provide $\ln(x)$ and not $\log_a(x)$. So, whenever you have to calculate $\log_a(x)$, you can use the formula to evaluate it using a normal calculator.

EXAMPLE:

- What is $\log_2(3)$?
- Solve the equation $2^x = 6$ and express the result as a natural logarithm.
- Show that for any a and b, the following is true: $\log_a(b)\log_b(a) = 1$.