

# Chapter 1

## The notion of functions

*Textbook Chapter 1*

### 1.1 The concept of functions

Although the concept of a function was invented a very long time ago, it is very easy today to gain an intuitive notion of what functions are because of their natural role in most computer and/or web-based applications, in engineering, and in economics, etc.

EXAMPLE 1A: ORDERING DIAPERS ON AMAZON

1 Diaper bag: \$45

When ordering  $n$  bags: Price =  $45 \cdot n + \text{tax} + \text{shipping}$   
Tax rate 8.5%  $\rightarrow 45 \cdot n \cdot 0.085 = \text{tax}$

$$\text{Price: } 45n + 0.085 \cdot 45n + 10 = 48.825n + 10 = p(n)$$

EXAMPLE 1B: BUYING TOMATOES AT SAFEWAY, SELF-CHECKOUT

Tomatoes: \$4/pound

Self checkout: weight  $\rightarrow w$  (pounds)

$$\begin{aligned} \text{Price: } 4w + \text{tax} &= 4w + 0.085 \cdot 4w \\ &= 4w(1 + 0.085) = 4.34w = p(w) \end{aligned}$$

EXAMPLE 2: BANK ACCOUNTS. CD accounts offer various interest rates depending on the amount you put in: for instance, as of 09/19/15, at Chase, for a 2-year fixed term deposit you get interests of:

- 0.15% per year for accounts under \$10K
- 0.25% per year for accounts between \$10K and under \$100K.

- 0.30% per year for accounts between \$100K and under \$250K.

The banker uses a computer function to tell you what your gain after 2 years will be as a function of your initial investment (cf. <https://www.bankofinternet.com/calculators/apy-interest-calculator>):

If  $x < 10,000$  Interest rate: 0.15%

Money in account after 1 year:  $x + 0.0015x = 1.0015x$

2 years:  $1.0015x + 0.0015 \cdot (1.0015x)$   
 $= 1.0015x (1 + 0.0015)$   
 $= 1.0015^2 x$

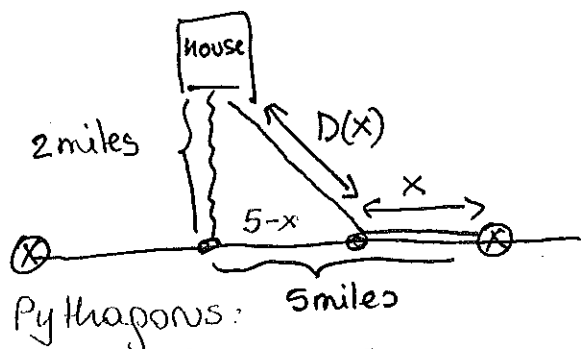
Gain =  $(1.0015)^2 x - x = [(1.0015)^2 - 1] x = 0.003x$

If  $10K < x < 100K$  : Interest rate 0.25%

Gain =  $[(1.0025)^2 - 1] x = 0.005x$

If  $100K < x$  Interest 0.30%  $\Rightarrow$  Gain =  $[1.003^2 - 1] x$   
 $= 0.006x$

EXAMPLE 3: INSTALLING CABLE TV (SEE PROBLEM 22 PAGE 105 OF TEXTBOOK).



Cost of installing along road: 500 \$/mile  
 off road: 700 \$/mile

Question: what is cost as function of  $x$ ?

$$\text{Cost} = \underbrace{\text{Cost off road}}_{700 \cdot D(x)} + \underbrace{\text{Cost along road}}_{500 \cdot x}$$

$$D(x) = \sqrt{4 + (5-x)^2}$$

$$\text{Cost}(x) = 700 \sqrt{4 + (5-x)^2} + 500x$$

## 1.2 Mathematical definitions

### 1.2.1 Definition of a function

DEFINITION: A function.

This definition is much more general than what we have seen so far, and covers a much wider class of functions, as for instance in the following examples: