ANSWERS

# Algebra Workshop 1: Simple manipulation of expressions

## 1 Manipulating negative signs

### 1.1 Removing brackets in an expression

RULE:

- Brackets preceded by a $+$ sign can just be removed

- Brackets preceded by a $-$ sign required some care: think of them as $-1 \times$ (expression). To remove the bracket, you need to distribute $-1$.

EXAMPLES:

- $(x^2 + 2x - 1) + (x + 3) = x^2 + 2x - 1 + x + 3 = x^2 + 3x + 2$

- $(x^2 + 2x - 1) - (x + 3) = x^2 + 2x - 1 - x - 3 = x^2 + x - 4$

- $-(x^2 + 2x - 1) - (x + 3) = -x^2 - 2x + 1 - x - 3 = -x^2 - 3x - 2$

IMPORTANT: Bear in mind which bracket corresponds to which sign. For example,

$$-[2x + 2 - (x + 1)] = -2x - 2 + (x + 1)$$

and not $-2x - 2 + (x - 1)$ (i.e. don't change *all* the signs automatically).

PRACTICE: Remove the brackets and simplify.

- $(x^3 + 3x^2 - 2) - (x - x^3) - (3x + 2x^2)$  $= 2x^3 + x^2 - 4x - 2$

- $-(x + 1) + (4x^3 - 2x + 1) - [-(x + 2) + (x^2 + 1) - (x^3 + 2x - 1)]$  $= 5x^3 - x^2$

- $3x + 2 - \{(x + 2) + (x^3 - 3x + 1) - [-(x - 2) + (2 - 3x^2)]\}$  $= -x^3 - 3x^2 + 4x + 3$

- $[3x + 2 - (x + 1)] - [2x + 1 - (3x^2 + 3x + 3)]$  $= 3x^2 + 3x + 3$

### 1.2 Adding brackets in an expression

RULE:

- You can always add brackets around any expression preceded by a $+$ sign without changing anything.

- If you add brackets around an expression preceded by a $-$ sign you have to change the sign of each term in the new bracketed expression.

EXAMPLES:

- $x^2 - 2x + 1 = (x^2 - 2x + 1) = (x^2 - 2x) + 1 = x^2 - (2x - 1)$

- $-x^3 + 2x^2 - 3x - 1 = -(x^3 - 2x^2 + 3x + 1) = -(x^3 - 2x^2 + 3x) - 1 = -(x^3 - 2x^2) - 3x - 1 = -(x^3 - 2x^2) - (3x + 1) = -x^3 + (2x^2 - 3x - 1) = \dots$

PRACTICE: Write these expressions at least 5 different ways using brackets in different places. Treat the various expression $(x - a)(x - b)$ in the second expression as "1 term" (i.e. do not expand them).

1

- $-2x^3 - 2x^2 + x - 4 \quad = \quad -(2x^3 + 2x^2) + (x - 4) = -(2x^3 + 2x^2 - x) - 4 \cdots$
- $-x(x-1) + (x-2)(x+1) - 2(1-x) + 1 \quad = \quad -\left[x(x-1) - (x-2)(x+1)\right] - \left[2(1-x) - 1\right]$

PRACTICE: Combine these expressions into a single bracket, written as $-[...]$, *and then* simplify inside this bracket (i.e do not simplify first).

- $-(x+2) + (x-1) = -[...] \quad = \quad -\left[(x+2) - (x-1)\right] = -\left[x+2-x+1\right] = -[3]$
- $(2x^2 + x - 1) - (2x^3 + x + 1) = -[...] \quad = \quad -\left[-(2x^2 + x - 1) + (2x^3 + x + 1)\right] = -\left[2x^3 - 2x^2 + 2\right]$
- $-(x+2)(x-2) - (x-1)(x+1) = -[...] \quad = \quad -\left[(x+2)(x-2) + (x+1)(x-1)\right] = -\left[2x^2 - 5\right]$

## 2 Expanding polynomials

VOCABULARY: Expanding a polynomial means to take whichever form it is currently in, and put it in the following form:
$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + ... + a_0$$

EXAMPLE: $(x+1)(x-2) + 3 = x^2 - 2x + x - 2 + 3 = x^2 - x + 1$. There, the largest value of $n$ is 2, and $a_2 = 1$, $a_1 = -1$ and $a_0 = 1$.

IMPORTANT: When dealing with polynomials with more than 2 factors multiplying each other, first multiply two factors and simplify that, then multiply the third one to the result. Note that it doesn't matter which two terms you first choose.

EXAMPLE:

$$
\begin{aligned}
(x-1)(2-x^2)(3x+1) \quad &= (2x - x^3 - 2 + x^2)(3x+1) \\
&= (-x^3 + x^2 + 2x - 2)(3x+1) \\
&= -3x^4 + 3x^3 + 6x^2 - 6x - x^3 + x^2 + 2x - 2 \\
&= -3x^4 + 2x^3 + 7x^2 - 4x - 2
\end{aligned}
$$

PRACTICE: Expand the same polynomial two different ways by multiplying two other terms first.

IMPORTANT: When dealing with polynomials containing brackets and factored polynomials inside these brackets, it's usually worth simplifying first inside the bracket.
IMPORTANT: Watch out for the signs!

EXAMPLE:

$$
\begin{aligned}
(x-1)[3 - (x-1)(x+2)] \quad &= (x-1)[3 - (x^2 + 2x - x - 2)] \\
&= (x-1)[3 - (x^2 + x - 2)] \\
&= (x-1)[3 - x^2 - x + 2] \\
&= (x-1)(-x^2 - x + 5) \\
&= -x^3 - x^2 + 5x + x^2 + x - 5 \\
&= -x^3 + 6x - 5
\end{aligned}
$$

PRACTICE: Expand the following polynomials. Be careful of the signs.

- $(x+1)(x-2) - (3x^2 + 1)(1-x) \quad = \quad 3x^3 - 2x^2 - 3$
- $-2x(x-1) + (2-x)(x+1)(3-x) \quad = \quad x^3 - 6x^2 + 3x + 6$
- $-3[2x(x^3 - 1) - 6x^2(x+1)] + 2x(x+1) \quad = \quad -6x^4 + 18x^3 + 20x^2 + 8x$
- $(4x+2)[(2x-1)(x+3) - 2] + (x+1)[3x(x-1) - 2x^2] \quad = \quad 9x^3 + 22x^2 - 13x - 10$

2