3.2.5 Quadratic functions summary

For a given quadratic function $f(x) = ax^2 + bx + c$

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3.3 Higher-order polynomials

Textbook section 4.6

3.3.1 Definition and examples

DEFINITION:

VOCABULARY:

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EXAMPLES:

In order to understand the behavior of these polynomials, let's start with studying the behavior of individual terms: functions which are simple powers of x.

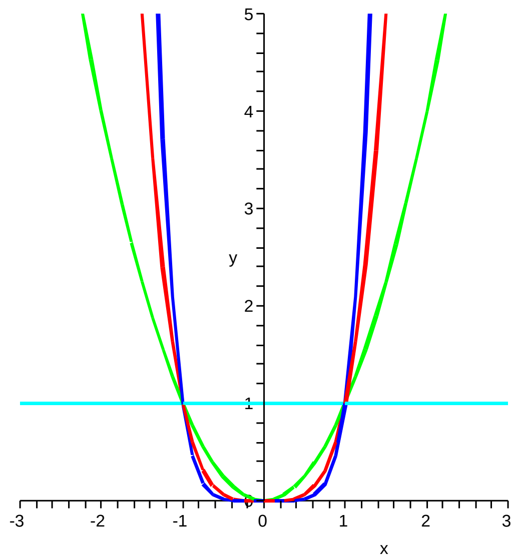
3.3.2 Functions of the kind $f(x) = x^n$ with n a natural number

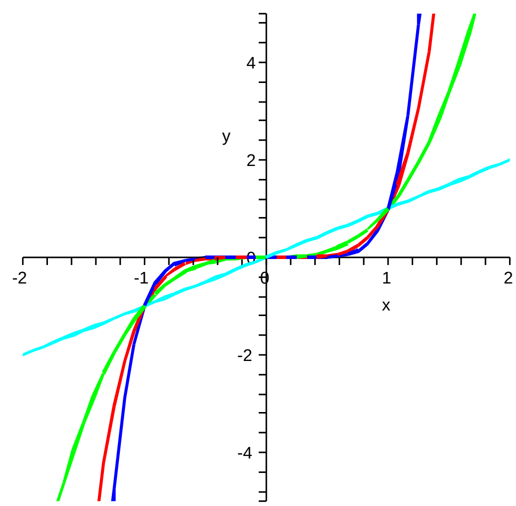
REAL-LIFE x^n FUNCTIONS: Power functions with integer powers arise naturally in geometrical problems. For example:

- The circumference of a square as a function of side length:
- The area of a square as a function of side length:
- The volume of a cube as a function of side length:
- The surface area of a cube as a function of side length:
- The circumference of a circle as a function of radius:
- The area of a circle as a function of radius:
- The volume of a sphere as a function of radius:
- The surface area of a sphere as a function of radius:

NOTE: Because we live in a three-dimensional world, a volume is always related to the cube of the meaningful lengthscale of the problem, and an area is always related to the square of the meaningful lengthscale. If we lived in a 4-D world ...

The shape of the graphs of functions of the kind $f(x) = x^n$ depends on whether n is an even or an odd number.





NOTE:

3.3.3 Functions of the kind $f(x) = ax^n$ with n a natural number

When the power is multiplied by a number a, note that

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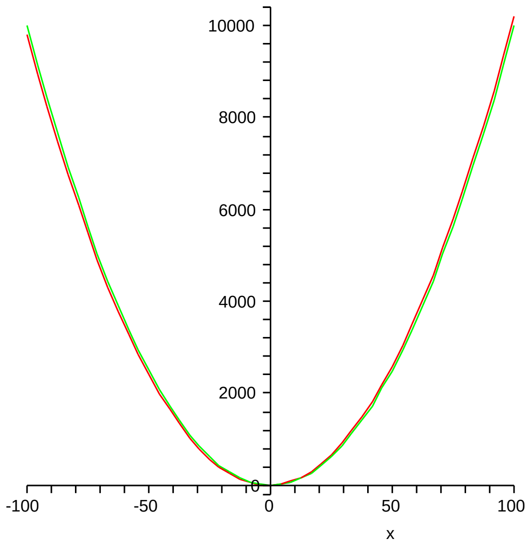
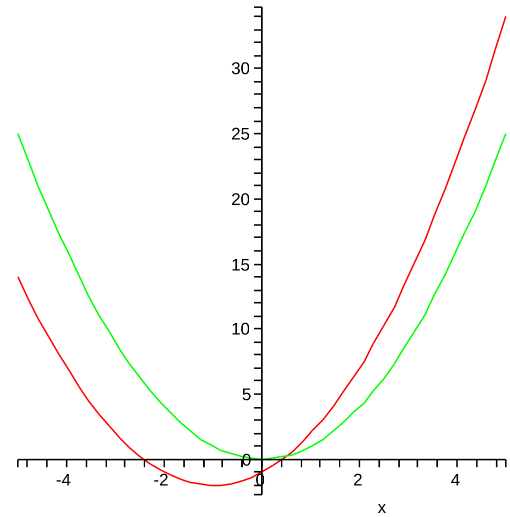
3.3.4 Approximations of polynomials for very large values of $|x|$

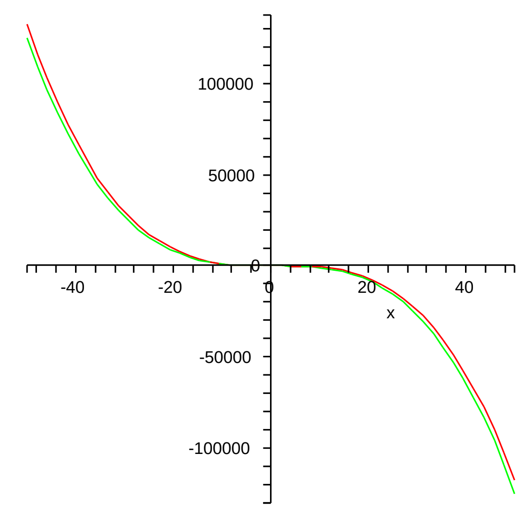
The overall shape of the graph of a polynomial function, when x is either very large and positive (x going to $+\infty$) or very large and negative (x going to $-\infty$), resembles that of the corresponding power function of the leading order term.

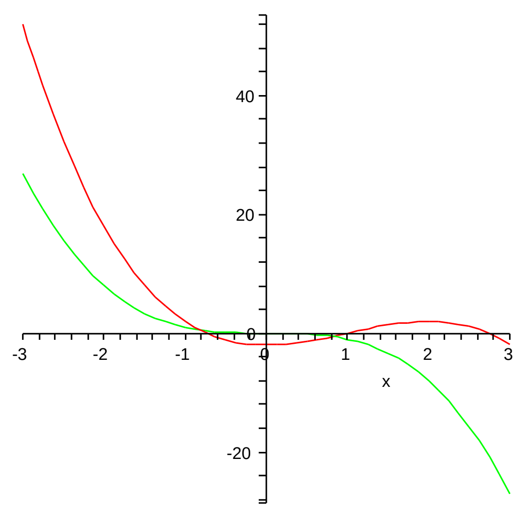
Example 1: $f(x) = x^2 + 2x - 1$





Example 2: $f(x) = -x^3 + 3x^2 - 2$





Conclusion:

3.3.5 Factored polynomials

In Chapter 1, we learned how to factor polynomial expressions. We now revisit the problem, and learn what the factored form of a polynomial function tells us about the graph of the function.

Formal definition of factored form:

Examples:

- $f(x) = -(2+x)(x+3)^3$

- $f(x) = (x-1)(2-x^2)$

- $f(x) = -2x(x^2-2x+1)(x+3)$

- $f(x) = x^3+2x^2+4x$

Signs tables for factored polynomials:

To create a signs table for a fully factored higher-order polynomial, simply follow the same method as for quadratic functions:

- Draw the table
- Write **all** the factors vertically on the left

- Write **all** the roots horizontally on the top (in the correct order)

- Draw vertical lines below each root

- Determine and write the sign of each factor; write zeros where appropriate e.

- Multiply the signs in each interval to determine the sign of the function.

Examples:

- $f(x) = -(2 + x)(x + 3)^3$

- $f(x) = (x - 1)(2 - x^2)$

- $f(x) = -2x(x^2 - 2x + 1)(x + 3)$

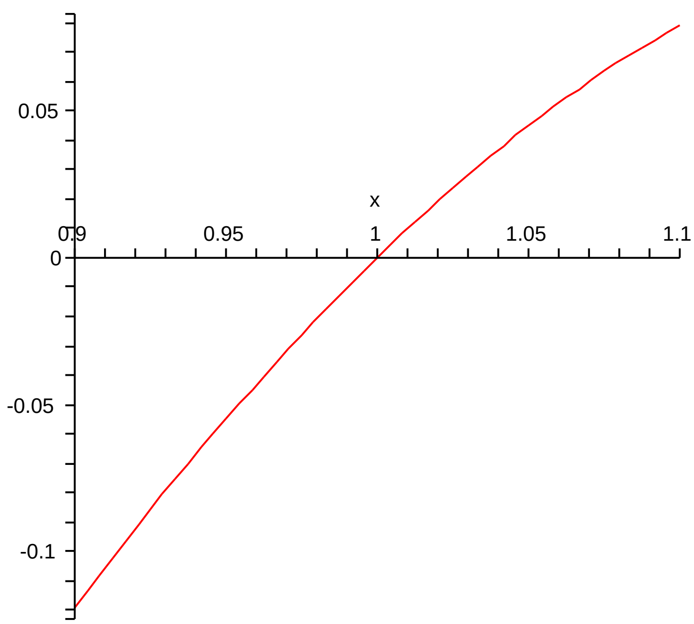
- $f(x) = x^3 + 2x^2 + 4x$

NOTE:

Behavior near a root

While the signs table typically gives you most of the information you need about the function, it is also useful to study the behavior of the function *in the vicinity* of a root, to double-check the signs table and sometimes to find out more about the function.

EXAMPLE 1: Consider the function $f(x) = (x-1)(2-x^2)$. Near $x = 1$, of course, $f(x)$ is close to 0 (because the $x-1$ term becomes very small). But what does it look like?



Since x is close to 1, let's see what happens if we plug $x = 1$ into *all the factors except* $(x-1)$. Then we get

When plotted on the same plot, the two functions are indeed very close to each other for x near 1.

NOTE: This can also help us check the signs table: the table says that $f(x)$ goes from negative to positive as x goes through 1. The line $y = x - 1$ also goes from negative to positive as x goes through 1.

EXAMPLE 2: Consider the function $f(x) = -2x(x^2 - 2x + 1)(x + 3)$. What does it look like near $x = 1$?

EXAMPLE 3: Consider the function $f(x) = -(2 + x)(x + 3)^3$. What does it look like near $x = -3$?

HOMEWORK: Now plot all 4 functions on graphing paper, and see that the analysis was correct.