2.6 The inverse of a function

Textbook section 3.6

2.6.1 Definition and examples

Definition:

As a result

Graphically:

- $y = f(x) = 3x + 2$:

- $y = f(x) = x^2$ (for $x \geq 0$):

- $y = f(x) = \sqrt{x-2}$ (for $x \geq 2$):

IMPORTANT NOTES:

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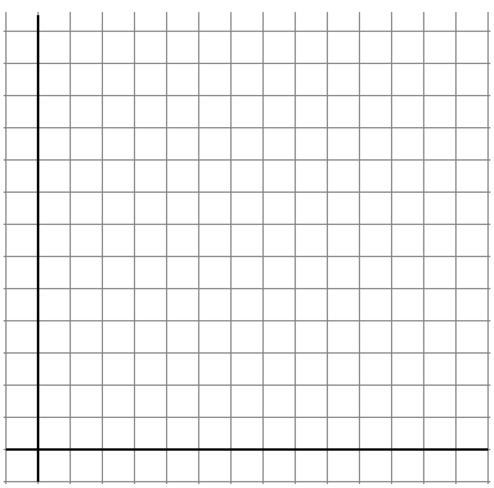
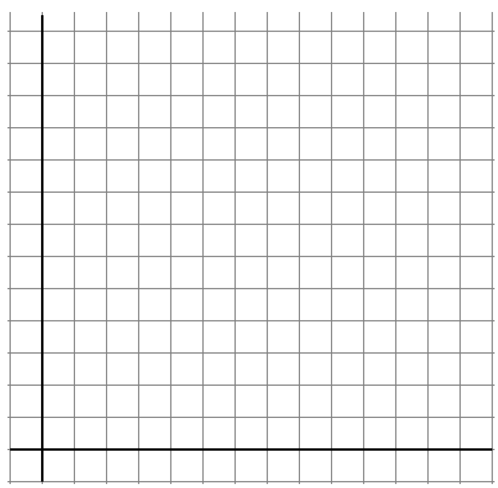
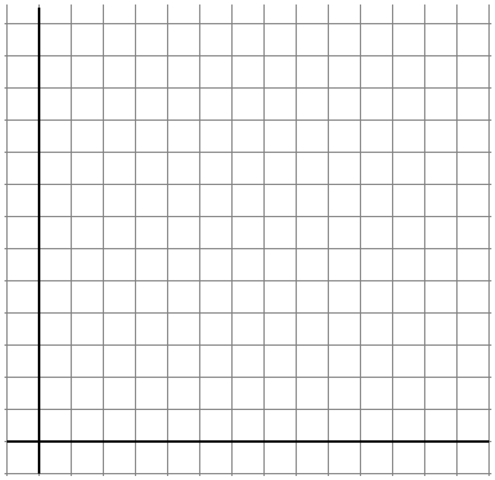
2.6.2 Graph of an inverse function and horizontal line test:

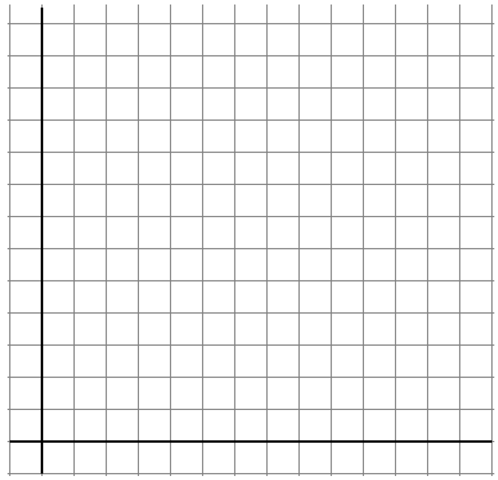
EXAMPLE 1: $y = f(x) = 3x + 2$





EXAMPLE 2: $y = f(x) = x^2$





So from these graphs we notice that:

NOTE: It may happen that through this process, the graph of the inverse does not satisfy the vertical line test: in that case, the inverse is not uniquely defined.

Horizontal line test: To verify that the inverse of a function is unique, we check that the function satisfies the horizontal line test:

When a function $f(x)$ does not satisfy the horizontal line test, we can often choose a smaller domain for which the inverse *is* unique.

Example: for the function $f(x) = x^2$, we saw earlier that the inverse of $f(x) = x^2$ is defined provided we select only the interval for which $x \geq 0$. In this interval, the function $f(x)$ does satisfy the horizontal line test.

2.6.3 More examples of inverse functions

2.7 Why are functions and operations on functions useful?

Not in textbook

Example: Why does it get harder to blow up a balloon?

Imagine trying to blow up a spherical balloon. Every 1 second, you exhale a volume V_0 of air into the balloon. How quickly does it grow?

Chapter 3

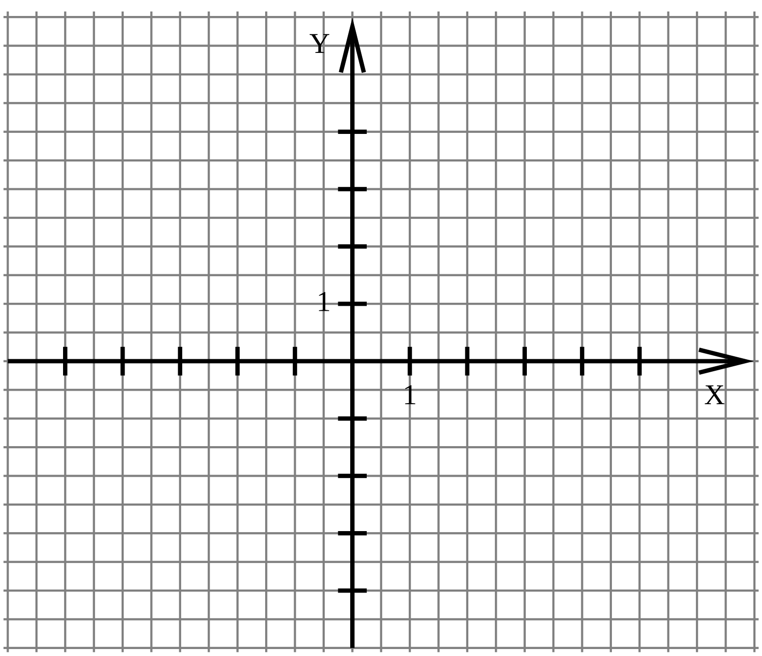
Polynomial functions

3.1 Linear functions

Textbook Section 4.1

DEFINITION:

GRAPH OF A LINEAR FUNCTION: The graph of $y = f(x) = ax + b$ is a straight line with *slope* a and $y-$intercept b



Some very cool examples of real-life linear functions.

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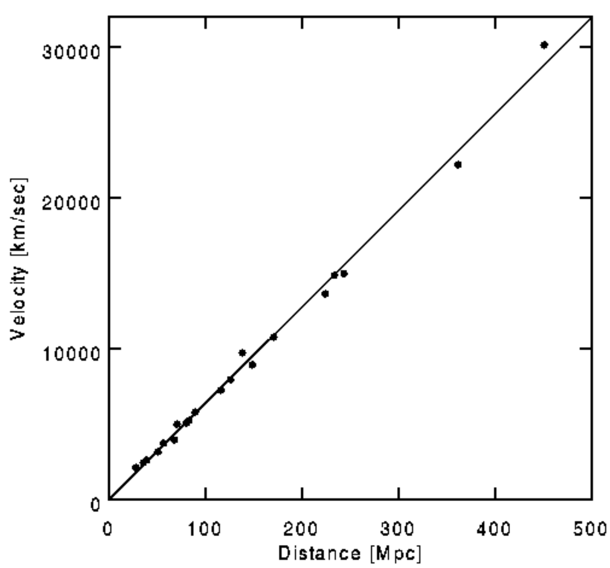
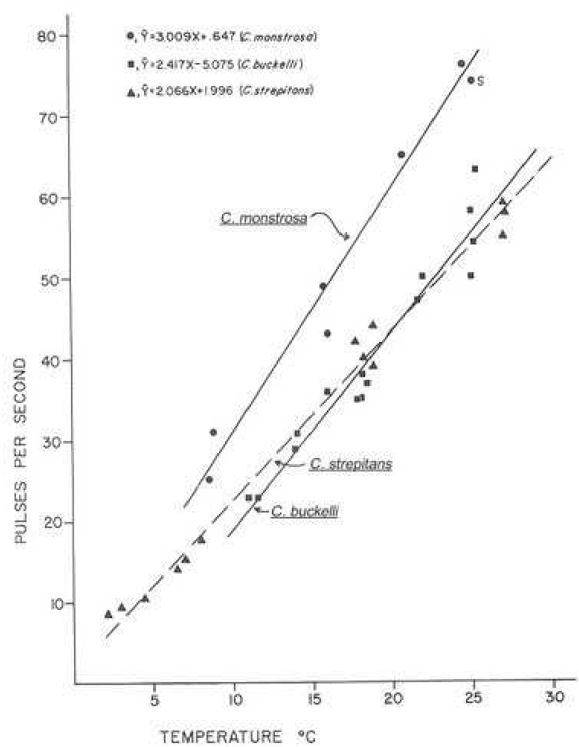




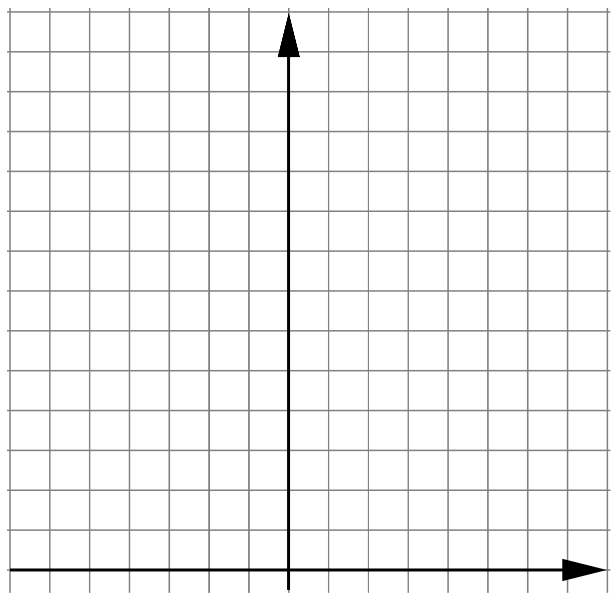
3.2 Quadratic functions

Textbook Section 4.2

3.2.1 Definition and basic $f(x) = x^2$ function

The general expression for a quadratic function is

The simplest example of a quadratic function is the function $f(x) = x^2$:



3.2.2 Deducing the graph of quadratic functions from the $y = x^2$ graph

In fact, the graph of all quadratic functions is a *parabola*. The exact shape and position of the parabola depends on the coefficients of the quadratic. In this section, we learn how to deduce the graph of a quadratic function from the graph of $y = x^2$, using the method of "completing the square".

EXAMPLE 1: $f(x) = x^2 - 2x + 3$.

Step-by-step: From $ax^2 + bx + c$

- Factor a in the first two terms:

- Complete the square in bracket using either of these formula: $x^2 + 2Kx + K^2 = (x + K)^2$ or $x^2 - 2Kx + K^2 = (x - K)^2$:

- Expand out again:

Then, to find the graph of the parabola, note that

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EXAMPLE 2: $f(x) = 3x^2 - 3x - \frac{1}{4}$.

EXAMPLE 3: $f(x) = -2x^2 - 3x + 1$.

3.2.3 Roots of quadratic functions

DEFINITION:

The roots of the quadratic function $f(x) = ax^2+bx+c$ are all the solutions of the equation $ax^2+bx+c = 0$. We saw in previous lectures that to find these solutions, we simply have to follow the rule:

- Calculate the discriminant $D = b^2 - 4ac$
- If $D < 0$ there are no solutions to the equation

- If $D = 0$ there is one solution to the equation, $x = -\frac{b}{2a}$
- If $D > 0$ there are two solutions $x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$

Where does this formula come from?

First, note that there are three types of possibilities for graphs of parabola:

Is there a way of knowing, from the expression for the quadratic, which case we have? YES!

3.2.4 A first introduction to signs tables

Signs Tables are an excellent tool to determine the *sign* of any polynomial function, and here we learn to use them for quadratics. NOTE THAT SIGNS TABLES CAN ONLY BE USED IF THE FUNCTION IS ALREADY BROKEN DOWN INTO ITS FACTORS. Then

- Draw the table
- Write **all** the factors vertically on the left
- Write **all** the roots horizontally on the top (in the correct order)
- Draw vertical lines below each root
- Determine and write the sign of each factor; write zeros where appropriate.
- Multiply the signs in each interval to determine the sign of the function.

EXAMPLE: $f(x) = 4(x - 1)(x + 2)$.

EXAMPLE: $f(x) = -2(1 - x)^2$.

EXAMPLE: $f(x) = (3 - x^2)$.