1.7 Absolute value

Textbook Section 1.2

1.7.1 Definitions and properties

Definition:

When taking the asbolute value of an expression, it is important to simplify the expression first.:
Examples:

Note that there are three important properties of absolute values:

-
-
-

Geometric interpretation: Absolute values can also be understood as the distance between two points on the real line.

Examples:

1.7.2 Manipulating absolute values

Important: For any expression E,

which means that in order to simplify an expression containing a term like $|E|$ you *must* first determine if (and where) E is positive and/or negative.

Examples:

1.7.3 Absolute values, inequalities and intervals

In the previous Lecture, we saw that inequalities represent intervals on the real line and are equivalent to the statement "x belongs to the interval". The same can be done with absolute values, but with much more care! However, if you remember that $|a - b|$ is the distance between a and b, things are a lot easier.

Examples:

1.8 Solving equations

Texbook Section 1.3

1.8.1 Variables, and domains

Equations are algebraic expressions which relate one (or more) unknown quantities (called *variables*) to known things (usually real numbers). The variables are often described by a letter like x, y, a, b, α, ξ (although there is no a-priori reason not to use more interesting characters such as).

A solution to the equation is a value of the variable for which the exression is true:

Depending on the equation, there may be no solutions, one solution, two solutions, ..etc.. and even sometimes an infinity of solutions.
Examples

Important: Before attempting to solve an equation it is important to determine first the *domain* of the variable.

Examples:

1.8.2 Linear equations

There are MANY different types of equations. The simplest kind is the **linear** equation for a single variable:

The solution of this equation is

There are many real-life situations which require the solution of linear equations...

Example: An employee at Yahoogle receives a startup bonus of $100,000, in addition to a monthly salary of $25,000. After how many months will he/she be able to afford his/her dream $1,000,000 home? (I know, this is a bit over-simplified...)

There are also many single-variable equations which may look complicated, but in fact reduce to a linear equation after some algebraic manipulations. Allowed manipulations are:

Examples: Equations that simplify to a linear equation:

1.8.3 Higher-order polynomial equations

Definition:

In this class, we will mostly study two kinds of polynomial equations:

- Quadratic equations ($n = 2$):

- Higher order equations ($n > 2$), already factored or easy-to-factor:

The latter case is then very easy to solve, noting that :

Examples:

The case of quadratic equations will be the subject of in-depth studies in a later lecture, but in the meantime note that:

- Whether this equation has solutions or not depends on the value of the **discriminant**

- Rule:
  - If $D < 0$:
  - If $D = 0$:
  - If $D > 0$:

Examples: