Chapter 6

Trigonometric functions

In this final Chapter we learn about the basic trigonometric functions, including the sine function and the cosine function, and others such as tangent, co-tangent, secant and co-secant. Note that we will mostly skip Chapter 6 in the textbook, except for 6.1 and 6.5.

6.1 Degrees and radians

Textbook Section 7.1

There are two major ways of measuring angles in geometry: in *degrees* and in *radians*.

The degree measure was introduced historically in astronomy to measure the displacements of stars, and is based on the fact that there are approximately 360 days in a year (well, there are in fact 365.25 days in a year, but 360 conveniently divides nicely by 2, 3, 4, 6, 10, 12, .., while 365.25 doesn't).

The radian measure is the one more commonly used in mathematics. It is based on the length of portions of a circle:

Based on this we have the correspondance:

To summarize, to go between radians and degrees and vice-versa,

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By convention, in mathematics we also define a direction to an angle:

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Since the circle wraps around, an angle is always defined up to a value of 2π:

6.2 Right-angle triangles and basic trigonometric functions

Textbook Section 6.1

6.2.1 Sine, cosine and tangent

Sine, cosine and tangent functions are usually defined through their association with right-angle triangles:

IMPORTANT CONSEQUENCES: From this diagram, we see that there are two very important formulae relating these three basic trigonometric functions to one another:

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6.2.2 Co-tangent, secant and cosecant

There are three more important functions to learn, defined as follows:

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These 6 functions altogether form the basic trigonometric functions you will need to know.

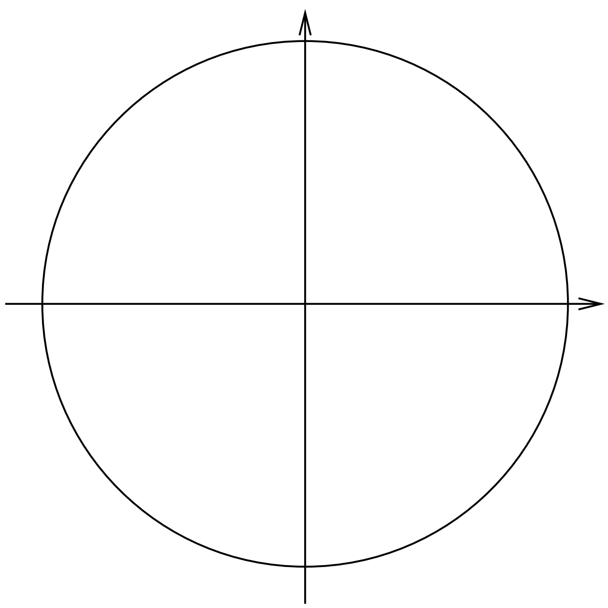
6.3 The unit circle, and the graphs of sine, cosine and tangent

Textbook Section 7.4

6.3.1 Construction of the unit circle

The unit circle is a wonderfully convenient way of *visualizing* the sine and cosine functions.

Definition:



Based on this, we can already deduce some particular values of the sine, cosine and tangent functions:

6.3.2 Sine and Cosine of important angles

In addition to $\pi/2$, π, $3\pi/2$ and 2π, there are 3 important angles for which you need to know the sine and cosine of:

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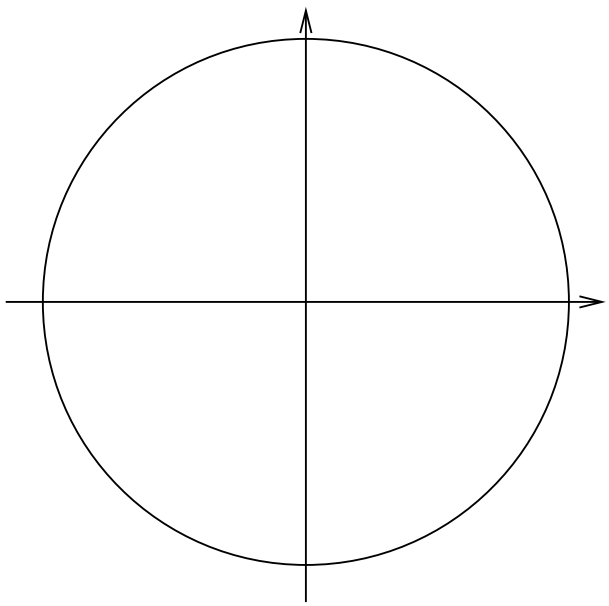
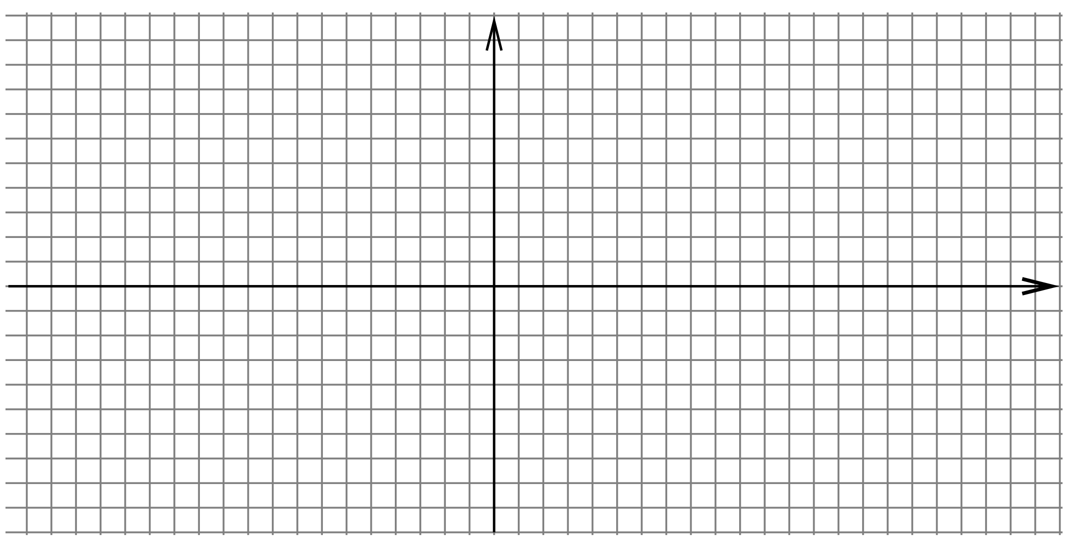
Based on the unit circle, we can now find the sine and cosine of many other angles:



Finally, we can use this information to plot the sine and cosine functions:



6.3.3 What can we deduce from the graphs of $\sin(x)$ and $\cos(x)$?

Based on the graphs of $\sin(x)$ and $\cos(x)$, we see that

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6.3.4 Periodicity

DEFINITION:

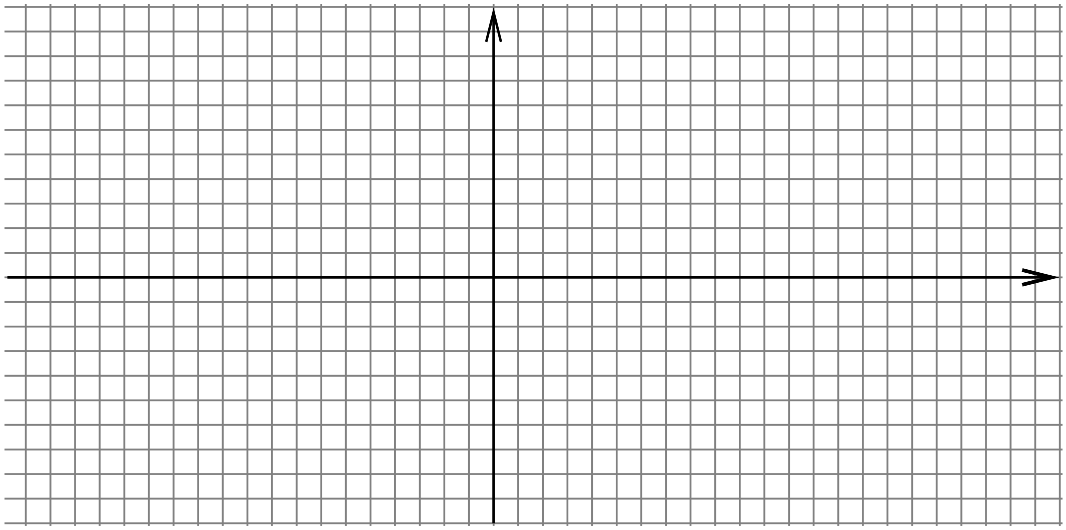
Based on the unit circle construction, and the fact

we conclude that

6.3.5 The graph of the tangent function

Textbook Section 7.7

The graph of the tangent function can be deduced from the graphs of the sine and cosine functions between $-\pi$ and π, and the fact that all basic trigonometric functions are 2π-periodic.



6.4 Oscillatory functions in general

Textbook Section 7.5

6.4.1 Examples of oscillations in nature

Many real phenomena are prone to *oscillations*. An oscillation is defined a very regular periodic behavior, and can usually be expressed in terms of sine and cosine functions. Natural examples of oscillations in nature are:

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Oscillations are usually characterized by 4 numbers:

- The mean:

- The amplitude of oscillation:

- The period of oscillation:

- The phase of oscillation:

6.4.2 Modeling oscillations

To model these various possible changes in oscillatory behavior, we can modify the basic trigonometric functions. Based on how the properties of the graphs of functions are changed, we see that

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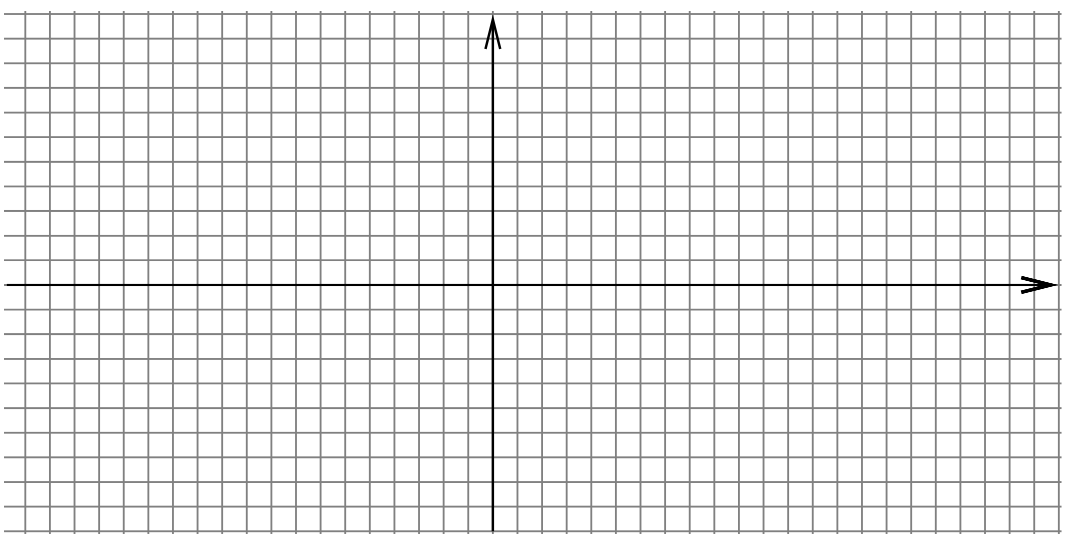
But how do we change the period? To answer this question, let's begin by graphing the function $f(x) = 2x$:



NOTE: It appears that the period of the function $\cos(2x)$ is π instead of 2π. Can we prove this?

6.4.3 General properties of the functions $f(x) = m + a\cos(cx + d)$ and $g(x) = m + a\sin(cx + d)$

As we saw in the previous sections, the functions $f(x) = m + a\cos(cx + d)$ and $g(x) = m + a\sin(cx + d)$ are oscillatory functions with the following properties:

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Note that the phase is often defined in different ways depending on applications, and also because sine and cosine themselves are actually the same function with a different phase (see previous sections).

Finally, the period of the oscillation can be deduced by asking the question: for which value of p is $f(x + p) = f(x)$ (and same for g). To find the period, we therefore have to solve the equation

This shows that the only number which changes the period of a function is the number multiplying x in the argument of sin or cos. More generally, we have just shown that

Example: What is the mean, period and amplitude of the following functions:

- $f(x) = 2 + 2\cos(2x + 2)$
- $f(x) = 3 + e\sin(\pi x)$
- $f(t) = \sin(2\pi t - 1)$