Chapter 1

Algebra Review

1.1 Numbers, representation on the line and intervals

Textbook Section 1.1

1.1.1 Definitions

- **Natural numbers** are numbers used, for example, to count people that are *present*. They form the set:

- **Integer numbers** are formed by considering the ensemble of all natural numbers and their negative version, as well as *zero*:

- **Rational numbers** are formed by taking the ratio of two integers:

- **Irrational** numbers are everything else...

What does that mean?

- In **decimal notation** *all* numbers can be written as:

  Example:

- For rational numbers,

- Irrational numbers are everything else...

Important:

-
-

1.1.2 Representation on the line

There is a one-to-one correpondance between points on a line, and real numbers:

1.1.3 Intervals and inequalities

An interval represents a continuous subset of the numbers. It is equivalent to a segment on the line:

An interval can also be represented by an inequality. The fact that there exist two types of inequalities ($\geq, \leq$ vs. $>, <$) implies that there are two types of intervals: *open* or *closed*:

-
-

Important: If the *end-point* is $\pm\infty$ then that end is always *open*.

Examples:

-
-

1.2 Integer exponents

Textbook Appendix B1

Definitions: Given a real number x and an integer n, then

-
-
-

All the properties of integer exponents can be deduced from these two:

-
-
-
-

Examples:

1.3 Polynomials, and manipulations of polynomials

A **polynomial** in the variable x can always be written in the form:

However, sometimes a polynomial can also be given in the form of products of factors, such as

To *expand* an expression (a polynomial) means to multiply out all the factors, and collect the terms according to the power of the variable in question.

Important: Note the three very important expression we derived:

-
-
-

When dealing with factors raised to a larger power, it is much easier to find the expanded expression by using "Pascal's Triangle":

1.4 Factoring Polynomials

Textbook Appendix B4

Factoring a polynomial is the opposite of expanding, it means to write the polynomial as a product of *factors*, in the form:

Factoring is not always easy .... There are a few basic factoring techniques, and then more advanced techniques (see later).

Basic factoring techniques:

- Common factor: Look for a common factor in *all* the terms of the polynomial

- Grouping: Group terms in the polynomials which may give rise to similar factors

- Difference of squares: This is a *MUST KNOW* standard formula for factorization...

- Two standard quadratic formulae: This is another *MUST KNOW* standard formula for factorization...

- Difference of cubes and sums of cubes: These can also be useful...

- Trial and error and a lot of “intuition”...

1.5 Fractions

Textbook Appendix B5

To use fractions, remember the rules:

-

-
-
-
-
-
-
-
-
-

With this, you can manipulate all expressions!
Examples:

1.6 Factoring Rational Expressions

Textbook Appendix B4 and B5

A rational expression is the ratio of two polynomials (either in factored form, or in expanded form):

However, sometimes it appears as the sum or difference of two expressions, in which case you first have to reduce them to the same denominator:

In order to factor rational expressions:

- factor the **numerator** and **denominator** separately
- use the simple agebra rules to simplify the resulting fraction

Examples: