Midterm

Name: _____

Calculators are not allowed.

Read all the questions before you start working on any of them. Start with the ones you are most comfortable with, and continue with the other ones later. Always double-check your answers. There is some scrap paper in the back; if you need more, please ask the instructor/proctor.

Relax, and do your best!

PROBLEM 1: SHORT QUESTIONS. In the following questions, you are merely asked to provide the answer. No justification is needed. You should not be spending more than 1 minute per question.

1. What is the equation of the line with slope 4 and $y-$intercept 3? _____

2. What is the equation of the line perpendicular to $y = 2x - 1$ which goes through the point $(1, 1)$?

_____

3. What is the center and radius of this circle? $(x + 8)^2 + (y - 5)^2 = 13$

Center: _____ , Radius: _____

4. Does the point $(-5, 2)$ lie on the circle of the previous question? _____

Given the functions $f(x) = 2(x - 1)^2$ and $g(x) = \sqrt{3(x + 1)}$

5. What is $f(2x + h)$? _____

6. What is the domain of $g(x)$? _____

7. What is $f \circ g(x)$? _____

8. What is $g \circ f(x)$? _____

9. Sketch the function $f(x) = -(x - 3)^2$

10, 11. Sketch the functions $g(x) = \sqrt{x-3} + 1$ and $h(x) = \frac{1}{x+2} + 2$

12, 13. Sketch the function $k(x) = (x-2)^3 + 1$ **and** its inverse on the same graph.

14. If $f(x) = x^3 + 2x + 1$, what is $f[f^{-1}(x+1)]$? _____

15. Complete the square for the expression $-x^2 + 2x + 3$: _____

Given the parabola $y = 2(x+2)^2 - 4$:

16. What are the coordinates of the vertex? _____

17. Does it open up or down? _____

18. What is the y−intercept? _____

19. What are the x−intercepts? _____

20. How many times does this parabola intercept the x−axis?

$f(x) = x^2 + 6x - 9$? _____

PROBLEM 2: FUNCTIONS AND INVERSES: Consider the function $f(x) = \frac{x+2}{x-3}$.

(a) What is the domain of $f(x)$? _____

(b) Draw a signs table for $f(x)$

(c) What is the behavior of $f(x)$ as x goes to $+\infty$ and $-\infty$?

_____

(d) Using this information, sketch $f(x)$.

(e) Calculate the inverse of $f(x)$. _____

(f) Verify that $f^{-1}[f(x)] = x$.

(g) What is the domain of $f^{-1}(x)$? _____

(h) Draw a signs table for $f^{-1}(x)$

(i) What is the behavior of $f^{-1}(x)$ as x goes to $+\infty$ and $-\infty$?

_____

(j) On the same graph as the plot for $f(x)$, plot f^{-1}. Clearly mark which is which.

PROBLEM 3. HIGHER ORDER POLYNOMIAL Consider the higher order polynomial function $x^3 - 4x^2 - 5x$.

(a) What is the behavior near $+\infty$ and $-\infty$?
When x tends to $-\infty$, $f(x)$ goes to _____
When x tends to $+\infty$, $f(x)$ goes to _____

(b) Factor the function: _____

(c) Determine the $x-$ and $y-$ intercepts

$x-$intercept(s): _____ $y-$intercept: _____

(d) Draw a signs table

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(e) Sketch the function

(f) Deduce the domain of definition of $\sqrt{x^3 - 4x^2 - 5x}$. Write the domain in interval notation.

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PROBLEM 4. APPLIED PROBLEM. A near-Earth asteroid approaches the Earth in a parabolic orbit defined by the equation $y = x^2 - 4$ (the Earth is at the origin of the coordinate system, E(0,0)). The units used here are millions of miles.

(a) Sketch the parabolic orbit. Make sure to indicate the $x-$intercepts and the $y-$intercept. Draw the two points where the asteroid is the closest to the Earth.

(b) The asteroid is a point $A(x, y)$ on the parabola. What is its distance between the asteroid and the Earth as a function of x only?

(c) What is the smallest distance of approach between the two objects? Hint: doing the problem directly is hard! Instead, consider minimizing the square of the distance, as a function of the square of x. A change of variable will be useful).