Homework 3

This homework is due in class on Monday 10/19/09

1 Course material

The definition of a function
Textbook Questions: Section 3.1: 6, 8, 10, 22 (domain only), 24 (domain only), 30, 32, 38, 44, 52, 56

The graph of a function
Textbook Questions: Section 3.2: 2, 4, 6, 18, 22, 24

Techniques in graphing
Textbook Questions: Section 3.4: 2, 4, 6, 8, 10, 18

2 Applied Problems

Cats (nearly) always land on their feet. However, whether they survive or not depends on how fast they are falling at the time they hit the ground. We will now calculate the velocity of a cat falling from a small tree (4m high). (No cats were harmed in the preparation of this question set).

The height of the cat h, as a function of time t, is given by the formula

$$h(t) = 4 - 4.9t^2 \text{ if } h(t) > 0$$
$$h(t) = 0 \text{ otherwise}$$

Question 1: What is the height of the cat at time $t = 0$?

Question 2: Calculate $h(t)$ for $t = 0.1$, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and $t = 1$. Graph the function $h(t)$.

Question 3: Calculate directly from the equation for $h(t)$ the time at which the cat falls onto the ground. you can verify your answer on your graph.

Question 4: To calculate a velocity, we use the formula

$$\text{velocity} = \frac{\text{distance travelled}}{\text{time it takes to travel this distance}}$$

What is the *approximate* velocity of the cat as calculated between its starting point at $t = 0$ on the roof and it's landing point (at $h = 0$ at a time you have to determine)?

Question 5: To get a more precise estimate for the velocity v of the cat at each point in time, we can use the following formula for the function $v(t)$:

$$v(t) = \frac{h(t + 0.1) - h(t)}{0.1}$$

Explain in words what this formula means, and how it relates to the definition of Question 4.

Question 6: Calculate $v(t)$ for $t = 0$, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and $t = 1$. Graph the function $v(t)$. What do you notice?

Question 7: Calculate $v(t)$ directly by substituting the expression for the function h in the expression for $v(t)$, and simplifying as much as possible. How does this explain your graph of Question 5?

Question 8: What is the velocity of the cat at the time it hits the ground? Note that the kitty survives falls unharmed with velocities below 10m/s.