Handout 2: Midterm Review Worksheet

This worksheet will help you prepare for the Midterm. Note that all of the material required for the Algebra Review test is also examinable for this upcoming midterm. If you make sure you master all of the material from both Reviews, you will do great!

Studying tips: Read each section. If you think you master the material, briefly look over the sample problems to convince yourself you know how to do them (if you're not sure how to do it, then try, and check your answers in the back,otherwise, quickly move on to the next section). If you don't think you master the material, go through the textbook and lecture notes about the material, make sure you understand the examples. Then do as many sample problems as you can, checking your answer in the back. The bottom line is: don't spend too much time on the points you know - make sure you spend most of your time studying for the points you're less familiar with.

If you have any difficulties, write them down, come to office hours to get help. Also, prepare some questions for the review on Monday.

The midterm will include all class and section material until the Wednesday before the midterm, **excluding inequalities**. The midterm will include *at least* 50% of questions which are extracted from the sample problems given in Handouts 1 and 2.

1 Equations of lines and circles

Equations of lines: See Section 1.6. You need to know, and know how to use the equation of a line, in particular

- how to calculate the slope of a line going through two points A(x_A, y_A) and B(x_B, y_B)
  $s = \frac{y_B - y_A}{x_B - x_A}$
- the point-slope formula
  $y - y_A = s(x - x_A)$ for the line with slope s going through the point A(x_A, y_A).
- the slope-intercept formula
  $y = sx + b$ for a line with slope s and $y-$intercept b
- the fact that if two lines are parallel they have the same slopes
- the fact that if two lines are perpendicular the product of their slopes is -1
- how to verify that a point is on the line

Sample problems: page 54-55, problems 15-36

Equations of circles: See Section 1.7. You need to know, know how to use and how to recognize, the equation of the circle of radius R centered on C(x_C, y_C):

$$(x - x_C)^2 + (y - y_C)^2 = R^2$$

For example, if you are given two points forming the diameter of the circle A(x_A, y_A) and B(x_B, y_B), you need to know how to find the coordinates of the mid-point C(x_C, y_C) and the radius (half the distance between the two points):

$$x_C = \frac{x_A + x_B}{2}\ ,\ y_C = \frac{y_A + y_B}{2}\ ,\ R = \frac{1}{2}\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$

You also need to know how to determine that a point is on a given circle or not.

Sample problems: page 68, problems 39-50, 53-58.

2 Definition and basic use of a function

See Section 3.1, 3.2 and 3.5

- You need to know how to identify that a rule is, or isn't a function
  Sample problems: page 141, problems 4-8
- You need to know how to determine that a graph is, or isn't the graph of a function (i.e. vertical line test)
  Sample problems: page 153, problems 5,6
- You need to know that every point on the graph of a function has coordinates $(x, f(x))$
  Sample problems: page 153-154, problems 1-4, 15,16
- You need to be able to determine the domain of definition of a given function
  Sample problems: page 141, problems 9-16
- You need to be able to evaluate functions at any given point (e.g. $f(2), f(x^2), f(x+h), f(a)$)
  Sample problems: page 142, problems 29-38
- You need to know the basic operations on functions (sum, product, quotient, ...)
  Sample problems: page 189, problems 1-6
- You need to know how to compose functions $f \circ g(x) = f[g(x)]$ and $g \circ f = g[f(x)]$
  Sample problems: page 190, problems 10-16

3 Graphs of basic functions

Graphs of basic functions: You need to know how to *sketch* the following functions without using a calculator. Sketching means "to draw something quickly, without too much attention to detail or perfect accuracy, but nevertheless represent any salient feature correctly".

- $f(x) =$ a constant (e.g. $f(x) = 2$, $f(x) = -\pi$)
- $f(x) = ax + b$
- $f(x) = |x|$
- $f(x) = x^2$, $f(x) = x^3$, $f(x) = x^4$, $f(x) = x^5$, etc...
- $f(x) = \sqrt{x}$
- $f(x) = \frac{1}{x}$, $f(x) = \frac{1}{x^2}$, $f(x) = \frac{1}{x^3}$, $f(x) = \frac{1}{x^4}$, etc...

See Section 3.4

Graphs of functions which are translated vertically and horizontally: Given the graph of a function $f(x)$, you have to know how to graph the functions

- $f(x) + a$: the graph is moved up or down (if a is positive or negative respectively) by an amount a
- $f(x + a)$: the graph is moved left or right (if a is positive or negative respectively) by an amount a

Graphs of functions which are reflected across the $x-$axis and $y-$axis: Given the graph of a function $f(x)$, you have to know how to graph the functions

- $-f(x)$: the graph is reflected across the $x-$axis
- $f(-x)$: the graph is reflected across the $y-$axis

Sample problems: page 179, 3-40

4 Inverse of functions

See Section 3.6.

- You need to know how to determine graphically that a function has an inverse (i.e. horizontal line test)
- You need to know the notation: the inverse of $f(x)$ is the function denoted by $f^{-1}(x)$.
- You need to know that $f^{-1}(x)$ DOES NOT mean $\frac{1}{f(x)}$.
- You need to be able to calculate the inverse of simple functions (i.e. solve the equation $y = f(x)$ for x.
  Example: $f(x) = \frac{2x-1}{3x+1}$. To calculate the inverse, solve $y = \frac{2x-1}{3x+1}$. The answer is $x = \frac{y+1}{2-3y}$. Then the inverse function is $f^{-1}(y) = \frac{y+1}{2-3y}$ or in other words $f^{-1}(x) = \frac{x+1}{2-3x}$.
- You need to know that the inverse f^{-1} applied to f yields x, and same for f applied to f^{-1}: $f[f^{-1}(x)] = f^{-1}[f(x)] = x$. You need to know how to use this fact to verify that the inverse you calculated is correct:

$$f[f^{-1}(x)] = f\left[\frac{x+1}{2-3x}\right] = \frac{2\frac{x+1}{2-3x} - 1}{3\frac{x+1}{2-3x} + 1} = \frac{\frac{2x+2-(2-3x)}{2-3x}}{\frac{3x+3+(2-3x)}{2-3x}} = \frac{2x+2-2+3x}{2-3x}\,\frac{2-3x}{3x+3+2-3x} = \frac{5x}{5} = x$$

- You need to know and understand the relationship between the graph of a function and the graph its inverse (i.e. they are mirror images with respect to the $y = x$ line). You need to be able to use that knowledge to find the graph of f^{-1} based on the graph of $f(x)$.

Sample problems: pages 203-204 problems 3, 4, 9-22

5 Quadratic functions

See Section 4.2. For a given function $f(x) = ax^2 + bx + c$ you need to know

- how to find the $y-$intercept (e.g. the $y-$intercept is c)
- how to complete the square to transform $f(x)$ into the vertex form $f(x) = a(x - x_V)^2 + y_V$.
  $f(x) = a\left(x^2 + \frac{b}{a}x\right) + c = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right) + c = a\left(x + \frac{b}{2a}\right)^2 - a\frac{b^2}{4x^2} + c$
- You need to know and understand from basic transformations of the $y = x^2$ parabola *why* $\left(x_V = -\frac{b}{2a}, y_V = f(x_V)\right)$ are the coordinates of the vertex of the parabola.
- based on the sign of a, whether the parabola opens up ($a > 0$) or down ($a < 0$)

- how to find $x-$intercepts (e.g. how to solve the equation $ax^2 + bx + c = 0$) using the discriminant method, see Handout 1.
- how to factor the quadratic depending on the discriminant (see Handout 1)
- how to draw a signs table for the quadratic, and how the signs table differs in the case $D = b^2 - 4ac$ is positive, zero and negative.
- how the signs table relates to the graph of the function f.

Sample problems: See Homework 4 and answer sheet.

6 Higher-order polynomials

See section 4.6. You need to know

- How to expand a polynomial, and recognize the leading-order term (the $a_n x^n$ term).
- That the behavior of the graph of the polynomial for very large x is the same as the behavior of the leading order term. How to deduce from that whether $f(x)$ goes to $+\infty$ or $-\infty$ as x goes to $+\infty$ or $-\infty$.
- How to recognize that a polynomial is fully factored or not
- How to factor it, if it is not fully factored (see Handout 1)
- How to draw a signs table for the polynomial
- How to deduce from the signs table what the shape of the graph is.
- How to find the behavior near a root.

Sample problems: pages 299-300 problems 27-44 (including, find the behavior near $+\infty$ and $-\infty$, and draw a signs table.)

7 Rational functions

See section 4.7. You need to know

- What the numerator and denominator are.
- How to reduce an expression to a rational function if necessary (see Handout 1)
- How to find the domain of definition of a rational function (i.e. exclude points where the denominator is 0).
- How to determine the behavior of the graph of the rational function for very large x (i.e. to look at the leading order terms in the numerator and in the denominator).
- What are, and how to find vertical, horizontal and oblique asymptotes.
- How to recognize that a rational function is fully factored or not
- How to factor it, if it is not fully factored (see Handout 1), and how to simplify it.
- What to do with "excluded points", points where both denominator and numerator have a root (i.e. to draw a circle where the point should be)
- How to draw a signs table for the rational function (remember the asymptotes).

- How to deduce from the signs table what the shape of the graph is. In particular, how to deduce what the shape of the graph near the asymptotes are.

Sample problems: pages 313-314 problems 9-34 (including, find the behavior near $+\infty$ and $-\infty$, and draw a signs table.)

8 Applied problems

The midterm will include one applied problem, similar to the ones given in the Homeworks and/or the problems discussed in class and in section. Remember that to solve applied problems:

- Identify what variables describe the problem. Give them names..
- Identify *all* the clues which relate the variables to one another. Write them as equations.
- Usually, you can then solve for one of the variables in one clue, then use that solution in another clue, or in what you are looking for, in order to make progress.

Solving word-problem is detective work! You need to use everything you know, and be creative!

Sample problems: page 269 problems 34-38, 42
Sample problems: pages 283-284 problems 49, 50, 55, 56, 57