Homework 3

January 23, 2020

1 Accoustic mirror

At the San Francisco Exploratorium, there is a rather amazing example of an accoustic mirror. The display is composed of two walls facing each other at two ends of a (usually very crowded) room, and each wall has a 2m-size half-sphere carved into it. When one person stands at the edge of the half-sphere and speaks into it, even very gently, another person standing in the same position but near the other half-sphere can hear the first person's voice very clearly. If, on the other hand, the first person turns around and speaks directly towards the second, it's usually impossible for the two people to hear each other across the noisy room.

Explain qualitatively how the accoustic mirror works, and why speaking directly to one another wouldn't. Hint: use ray theory and assume that a half-sphere is not too different from a parabola for simplicity. Think (qualitatively) about the amplitude equation. Note that a parabola has the property that parallel rays coming in from infinity all focus in the same point (called the focal point). Similarly, rays emitted from the focus end up going to infinity on parallel lines, after reflection on the mirror. See Wikipedia article on Parabolic Reflectors for instance.

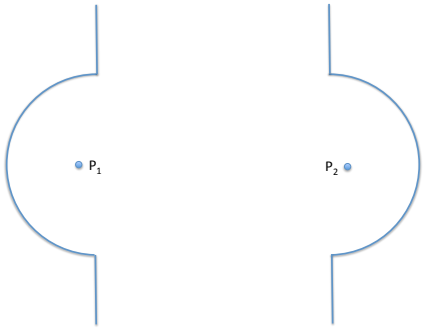

Figure 1: Schematic of the accoustic mirror. The two people are sitting at the two foci of the mirrors at P_1 and P_2, talking to each other while facing the mirrors, can hear each other across a crowded, noisy room.

2 Diffraction of sound waves

Consider a 2D uniform medium (with constant sound speed). Then, consider a small region where the sound speed is slightly larger, i.e. by writing that

$$c_s(X, Y) = c_0 + (\Delta c) \exp\left[-\frac{(X^2 + Y^2)}{2}\right] \tag{1}$$

where $\Delta c > 0$.

What will happen to the paths of waves coming from $Y \to +\infty$, along wavevectors that are initially along $\boldsymbol{e}_y$, as they approach the sound speed anomaly? A qualitative answer is sufficient. Sketch some representative ray paths. What happens if the sound speed anomaly Δc is negative? Sketch the ray paths in that case.

3 Waves in the Diskworld

If the world were a disk, and we lived in 2D, what would be rate of decay of the amplitude of sound away from a *circular* loudspeaker ?

4 Energy conservation

Prove equation (2.48) of the notes on energy conservation.