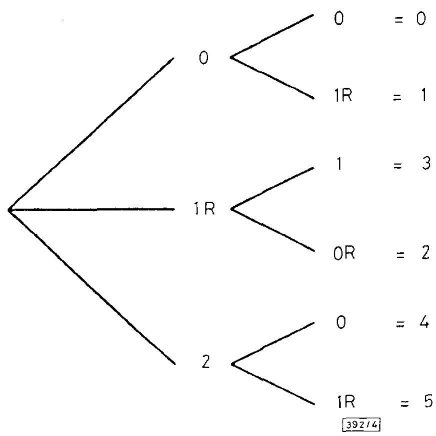

Fig. 4 *Hybrid tree structure*

If the sub-band tree is composed of layers of M-ary and N-ary branches, the codeword generation works in a similar way. Each digit for the codeword is in modulo-m for the layer corresponding to the M-ary branch and it is in modulo-n for the layer corresponding to the N-ary branch. Suppose the first branch is ternary, and it is followed by binary branches at each node. This time, the code generation is as follows:

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \mid 0 \\ 0 \mid 1 \\ \hline 1 \mid \\ 1 \mid \\ \hline 2 \mid \\ 2 \mid \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \mid 0 \\ 0 \mid 1 \\ \hline 1 \mid 1 \\ 1 \mid 0 \\ \hline 2 \mid 0 \\ 2 \mid 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \\ 4 \\ 5 \end{bmatrix} \qquad (4)$$

Visually, this corresponds to the tree in Fig. 4. When different number of branches exist at the same layer in the decomposition, the calculation of the frequency order is carried out separately for each branch.

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Least-squares multirate FIR filters

R. Manduchi and P. Perona

Indexing terms: FIR filters, Digital filters

The authors propose a new least-squares design procedure for multirate FIR filters with any desired shape of the (band-limited) frequency response. The aliasing, inherent in such systems, is implicitly taken into account in the approximation criterion.

Introduction: The multirate implementation of FIR filters (see Fig. 1), introduced by Rabiner and Crochiere [1], leads to reduced computational complexity. In fact, the samples at the output of the FIR filter $g(n)$ that are deleted by the M-fold sampler do not need to be computed, and the null-valued samples introduced by the M-fold interpolator do not contribute to the convolution operated by the FIR filter $h(n)$. Only the case of brick-wall frequency response was considered in [1], and the design technique was inspired by minimax criteria.

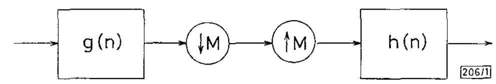

Fig. 1 *Multirate implementation of filter*

We propose a least-squares criterion for the design of multirate FIR filters, to approximate the spectral shape of any desired prototype $d(n)$ (assuming that the necessary band-limiting conditions are met i.e. that the spectral support of $d(n)$ has a length $<2\pi/M$). The resulting system is linear periodically time-invariant (LPTV [2]), and it is characterised by the M impulse responses $\{t^{(i)}(n+i), 0 \le i < M\}$, corresponding to the M inputs $\{\delta(n+i), 0 \le i < M\}$. The fact that the impulse responses differ from each other is usually referred to as the aliasing effect. The least-squares criterion introduced in this Letter makes for the joint reduction of the approximation error and of the inherent aliasing.

Theory: We only consider the case $M = 2$ (definitions and results are extended straightforwardly to the case of higher M). Define the polyphase components [2] of $g(n)$ as

$$g_0(n) = g(2n) \qquad g_1(n) = g(2n+1)$$

It is easily shown [2] that

$$t^{(0)}(n) = h * \bar{g}(n) \qquad t^{(1)}(n+1) = h * \bar{g}_1(n)$$

where $\bar{g}_0(n)$ and $\bar{g}_1(n)$ are obtained by interleaving $g_0(n)$ and $g_1(n)$ with null-valued samples.

We propose the following design criterion: given the kernel $d(n)$ to be approximated, find the filters $g(n)$ and $h(n)$ with given length N_g and N_h respectively, which minimise the approximation error ε^2, defined as

$$\varepsilon^2 = \frac{\|t^{(0)}(n) - d(n)\|^2 + \|t^{(1)}(n) - d(n)\|^2}{2} \qquad (1)$$

The term ε^2 implicitly accounts for both the approximation quality and the aliasing. In fact, if ε^2 is small, we may expect both impulse responses of the system to be 'close' to $d(n)$, and therefore 'close' to each other. More precisely, the following upper bound holds:

$$\|t^{(0)}(n) - t^{(1)}(n)\|^2 \le 2(\varepsilon^2 + \|t^{(0)}(n) - d(n)\| \|t^{(1)}(n) - d(n)\|)$$

No simple closed form solution can be found to the minimisation problem, since the error ε^2 in eqn. 1 is composed of quadratic forms of bilinear expressions in $g(n)$ and $h(n)$. A standard procedure in such cases is based on iterative minimisation [3]. Our iterative algorithm is briefly outlined in the remainder. Vectorial notation is used for sequences: a sequence $x(n)$ is represented by a column vector $\mathbf{x}$ whose entries are the samples of $x(n)$. The symbol ^{T} stands for vector/matrix transposition. We start from an initial guess of $g_0(n)$ and $g_1(n)$, and then iterate through the following two steps:

Optimisation of $h(n)$ for fixed $g_0(n)$, $g_1(n)$: Let $\bar{\mathbf{G}}_0$ and $\bar{\mathbf{G}}_1$ be the Toeplitz matrices representing the filtering with $\bar{g}_0(n)$ and $\bar{g}_1(n)$, respectively. Then

$$\varepsilon^2 = \frac{(\bar{\mathbf{G}}_0\mathbf{h} - \mathbf{d})^T(\bar{\mathbf{G}}_0\mathbf{h} - \mathbf{d}) + (\bar{\mathbf{G}}_1\mathbf{h} - \mathbf{d}_+)^T(\bar{\mathbf{G}}_1\mathbf{h} - \mathbf{d}_+)}{2}$$

where $\mathbf{d}_+$ is the vector representing $d(n+1)$. Hence, ε^2 is minimised for

$$\mathbf{h} = (\bar{\mathbf{G}}_0^T\bar{\mathbf{G}}_0)^{-1}(\bar{\mathbf{G}}_0^T\mathbf{d} + \bar{\mathbf{G}}_1^T\mathbf{d}_+)$$

Optimisation of $g_0(n)$ and $g_1(n)$ for fixed $h(n)$: Let $\mathbf{H}$ be the Toeplitz matrix representing the convolution with $h(n)$, and let $\mathbf{U}$ be a matrix obtained by interleaving the rows of a suitably sized identity matrix with null-valued rows. Then

$$\varepsilon^2 = \frac{(\mathbf{HUg}_0-\mathbf{d})^T(\mathbf{HUg}_0-\mathbf{d})+(\mathbf{HUg}_1-\mathbf{d}_+)^T(\mathbf{HUg}_1-\mathbf{d}_+)}{2}$$

Error ε^2 is minimised for

$$\mathbf{g}_0 = (\mathbf{U}^T\mathbf{H}^T\mathbf{HU})^{-1}\mathbf{U}^T\mathbf{H}^T\mathbf{d}$$
$$\mathbf{g}_1 = (\mathbf{U}^T\mathbf{H}^T\mathbf{HU})^{-1}\mathbf{U}^T\mathbf{H}^T\mathbf{d}_+$$

Since the error ε^2 does not increase at any iteration and is bounded from below by zero, we are guaranteed to converge to some minimum of ε^2. However, the minimum may be just local, and it may be useful to run the algorithm several times with different starting points, choosing the solution that gives the smallest ε^2.

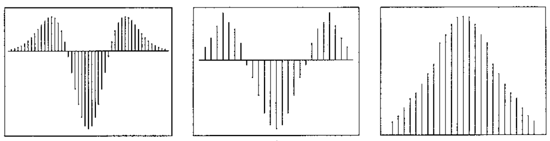

Fig. 2 *Kernel d(n) to be approximated and filters g(n) and h(n) minimising* ε^2

a Kernel to be approximated
b Filter *g(n)*
c Filter *h(n)*

A design example: We have tested the proposed design technique for a kernel *d(n)* shaped as the second derivative of a gaussian function, a filter widely used in computer vision (see Fig. 2). The standard deviation σ was set to 10 and the length of *d(n)* was 47 samples. The design parameters were: $M = 4$, $N_g = N_h = 25$. The multirate implementation thus requires approximately four times fewer elementary operations per input sample than the direct implementation of *d(n)*. The starting point for the iterative optimisation was a constant sequence *g(n)*.

To evaluate the multirate system's performance, we may define the signal to approximation noise ratio

$$SNR = \frac{\|d(n)\|^2}{\varepsilon^2}$$

and the signal to aliasing ratio:

$$SAR = \frac{\|d(n)\|^2}{\max_{i,j}\{\|t^{(i)}(n) - t^{(j)}(n)\|^2\}}$$

In our case, we obtained $SNR = 28.3$dB and $SAR = 25.8$dB.

Conclusion: The multirate implementation of band-limited FIR filters leads to the reduction of the computational weight. We have presented a novel least-squares technique to design multirate FIR filters for any shape of the (band-limited) desired frequency response. The technique is based on temporal domain approximation, and the error criterion accounts for both good approximation and aliasing.

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*N*th-order allpass voltage transfer function synthesis using CCII+s: Signal-flow graph approach

C. Acar

Indexing terms: Current conveyors, Active filters

A general synthesis method is given for the realisation of an *n*th-order allpass voltage transfer function by the active RC circuit containing a minimum number of capacitors and at most *n*+1 current conveyors. All the current conveyors are positive, and *n* of them act as voltage followers. This makes the proposed circuit simple and attractive.

Introduction: High-order allpass filters are widely used in crossover systems, and a broad class of transfer functions e.g. the lowpass, highpass, bandpass and bandreject characteristics can be implemented as the parallel combination of two allpass filters [1]. Although second generation current conveyors are functionally flexible and versatile active components [2], little work has been carried out towards the realisation of high-order allpass filters using current conveyors [3 – 5]. The purpose of this Letter is to give a synthesis method for the realisation of any *n*th-order allpass voltage transfer function by an active-RC circuit using only resistors, capacitors and second generation current conveyors with positive gains (CCII+s). The proposed method is based on realising the allpass transfer function by a signal-flow graph and then obtaining, from the graph, the active-RC circuit involving CCII+s.

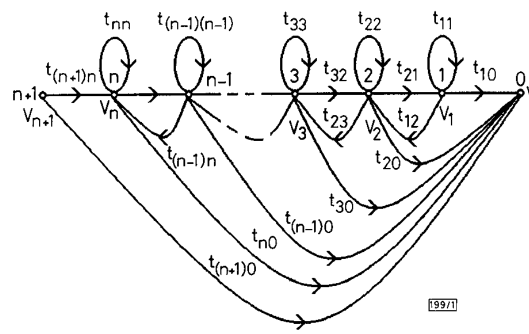

Fig. 1 *Signal-flow graph model realising T(s) of eqn. 1*

Realisation procedure: Let the *n*-th order allpass voltage transfer function be expressed as

$$T(s) = \frac{D(-s)}{D(s)} = \frac{(-1)^n a_n s^n + (-1)^{n-1} b_{n-1} s^{n-1} + \cdots + b_2 s^2 - b_1 s + 1}{b_n s^n + b_{n-1} s^{n-1} + \cdots + b_2 s^2 + b_1 s + 1} \quad (1)$$

where the denominator polynomial *D(s)* is a Hurwitz polynomial with positive real coefficients. This function can be represented by the signal-flow graph shown in Fig. 1 if the following values are assigned to the branch transmittances:

$$\begin{aligned}
t_{ii} &= 1 - k_i\left(s + \frac{b_{i-1}}{b_i}\right) & i &= 1, ..., n \\
t_{(i+1)i} &= k_i \frac{b_{i-1}}{b_i} & i &= 1, ..., n \\
t_{(i-1)i} &= k_i s & i &= 2, ..., n \\
t_{i0} &= 2(-1)^{i-1} & i &= 1, ..., n \\
t_{(n+1)0} &= (-1)^n & &
\end{aligned} \quad (2)$$

where $b_0 = 1$, and k_is, $i = 1, 2, ..., n$, are independent parameters introduced to the design. These transmittance values are obtained from the signal-flow graph [6] by modifying its transmittance values according to the elementary transformations [7]. It may be easily verified, using the well known Mason's gain formula, that the signal-flow graph in Fig. 1, with the above transmittance values, realises the *T(s)* of eqn. 1; i.e. the graph gain from the input node *n*+1 to the output node 0 is equal to *T(s)*.